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July 22, 1998

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Program Officer Richard L. Lau ONR 311
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Re: Technical Progress report for ONR grant # N00014-98-1-0514,
Yu Chen, Principal Investigator.

Dear Dr. Lau:

Enclosed please find three copies of the technical progress report for the research project entitled "Numerical Solution of Acoustic Inverse Scattering Problems" under the direction of Professor Yu Chen.

If I can provide any additional administrative information please feel free to contact me at the above referenced address.

Thank you for your interest in and support of our research programs.

Sincerely,

Joseph Hayes

Joseph Hayes
Fiscal Officer

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Numerical Solution of Inverse Scattering Problems

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Part I: Progress Report

Since the beginning of this year, the main activity of my research has been in the following two areas: to make the back propagation stable, and to design a frequency-local but space-global algorithm for one-dimensional inverse scattering problems. These two projects are being completed. The first one is an joint effort with V. Rokhlin whereas the second with Mr. Y. Xiang, a graduate student at the Courant Institute, currently support by the ONR grant during the summer.

Back Propagation: In a scattering problem, the measurement is the scattered wave naturally propagated outward from the scattering body. The inverse scattering problem, on the other hand, is in essence to propagate back the measurement into the scatterer where we wish to recover relevant parameters such as density. Mathematically, this back propagation of information is often formulated as certain initial value problems. The solution of these initial value problems is critical for inversion procedures such as layer stripping; it has also been used to solve the forward scattering problems when back scattering is not negligible. It is well-known that in two and higher dimensions, none of these initial value problems is well-posed which means that when implemented numerically, the procedure is unstable – the numerical solution usually blows up. This is one of the fundamental causes to instability in the layer-stripping approach in two and higher dimensions.

For acoustic scattering, the initial value problem for the back propagation is formulated as Riccati equations. Our goal here is to make this process stable and, if this can be done, to further make layer stripping stable for two dimensional problems.

The cause of instability is the so-called evanescent waves – modes that are high-frequency in spatial directions perpendicular to the direction of back propagation. These modes grow or decay rapidly in the direction of back propagation. The standard and seemingly attractive approach is to separate and cut off the nuisance evanescent modes from propagating ones – components of the wave that carry information and that don't decay. The problem with this approach is that no one knows how to make the separation.

We have indeed found a procedure that ensures a stable computation of the back propagation. Our idea is not to separate and cut off at all. It turns out that there is an initial value problem which is the dual of the original one for back propagation, and which is stable in the direction of the back propagation. Therefore, the dual problem is first solved and then used to combine with the initial value of the original problem to produce the solution of the back propagation. This approach is natural and no human intervention such as artificial cut-off is required. Our numerical experiments show that all propagating modes obtained this way are accurate.

Having made it stable, we are applying this back propagation procedure to layer-stripping calculation for the two-dimensional inverse scattering problem. Preliminary results of stable numerical inversion via layer stripping were presented at the AMS-IMS-SIAM summer research conference on July 9 at Mt. Holyoke, MA.

Frequency-local, Space-global Method: For one-dimensinal inverse scattering problem, the established method is layer stripping. There are several reasons for us to make an effort to design and analyze a different approach.

(i) Layer stripping is a frequency-global, space-local method: each time a layer is recovered (or removed, stripped), an integration is required over the entire k-space, or the frequency space. In an ideal situation where we have measurements over a wide range of frequencies, the k-space integration is a well-defined procedure. In a more realistic environment, however, the medium and high-frequency parts of the measurements for example are missing. Layer-stripping method, being frequency-global, suffers and usually breaks down whenever part of the spectrum is not available. Its strong dependence on the global properties of certain functions in the k-space is an obvious liability to the method;
(ii) Efficient implementation of the layer-stripping approach relies on the so-called trace formula which actually carries out the global integration in the k-space. The design of the trace formula requires and exploits the analyticity of certain functions of k in the upper half of the complex k plane. In addition, the algebraic process for the derivation of the trace formula requires a high-frequency asymptotic analysis which make it essential that the differential equations be explicitly given in close forms. These two requirements can be easily met in the model case of the scalar Helmholtz equation, but are too delicate to be satisfied in a more realistic setting where it is substantially more difficult to exploit the fragile structure of analyticity (examples are the scalar Helmholtz equations in polar coordinates) or to write down explicitly the differential equations (examples include inversion of the density and speed of sound simultaneously in a layered medium with a point source where offset measurements are required);
(iii) The third reason is more subtle. A scatterer that is simple in structure in the sense that as a function it can be specified by only a few parameters may generate extremely complicated patterns in the measured back-scattered field as a function of k. For example, a scatterer function that is simple but large in magnitude represents a high inhomogeneity and gives rise to strong scattering which translates to "rich" structures in the measurement. However, as we have discovered, the structures are not actually rich but highly correlated and inter-dependent. This means a limited measurements in the low-frequency range are sufficient to determine the scatterer function. Unfortunately, a frequency-global method fails to utilize the local structures and their correlations; it treats and weights each segment of the measurement spectrum equally, and thus needs to sample substantially more measurement data to adequately resolve the "rich" structures within them. In other words, a frequency-global method is inefficient in the use of data.

We have design and analyzed a frequency-local space-global approach that eliminates all the above problems associated with a frequency-global method. Our algorithm, being frequency-local, eliminates the problem of a global operation in the k-space; it does not rely on the use of delicate construct such as trace formula; it recovers with a few samples of the measurements a simple scatterer which generates strong scattering. Moreover, we have established convergence and stability analysis for this approach.

Other Activities: I have been collaborating with Professor G. Bao of University of Florida at Gainesville to analyze the recursive linearization method [3]; a paper was submitted to SIAM Journal of Math. Anal. for publication [1]. I have been working with a graduate student, Mr. L. Zhu, during the summer to experiment with the ray tracing method as a candidate for the fast forward modeling in the recursive linearization calculation.

Part II: Proposed Program of Research

I don't wish to deviate from what I proposed, early this year, to accomplish. Several months have passed and only a few pieces of this intricate puzzle of inverse scattering have been revealed. Therefore please let me reiterate the points I made before.

Recently we discovered several inversion methods (see [2]–[5]) which for the first time stably and accurately solve the fully nonlinear acoustic inverse scattering problem in two dimensions. Since then, we have developed two computer codes for the accurate numerical simulations of

(i) Sound soft and hard scattering problems in two dimensions;
(ii) Fully elastic scattering, in both frequency and time domain, for layered liquid and solids in three dimensions.

With these existing methods and tools, we intend to undertake the following investigations.

Solids in Water: The fully elastic scattering model can be adapted to study scattering of solids in water. The purpose of this activity is not to reconstruct the shape or internal structure of the solid but to characterize the type of the solid from offset measurements (when source and receiver are not co-located).

Several Applications: There are three areas of application of our inverse scattering methods. i) Obstacle inverse scattering; effort is needed to improve resolution which is now unacceptably low. It's applications range from detection of underwater mines to optimal design for fiber optics. ii) Electrical impedance imaging; although only limited number of parameters can be recovered, once this is done in a stable way, they provide valuable information of the object being imaged. iii) Apply our one-dimensional stable trace method (see [5]) to various inverse problems in layered media: inverse electromagnetic scattering, inverse elastic scattering, inverse acoustic scattering with density variation. This area has not been treated properly; I will design fast and accurate inversion algorithms for all these cases, and to complete their convergence analyses.

Fast Algorithms: Implement a fast and high order direct solver, based on volume integral equations, for the Helmholtz equation. Besides other applications, such an algorithm will be directly used as the forward solver (of the Helmholtz equation) in the inversion to accelerate the inverse solver. I expect the resulting procedures to be adequately fast for most inverse scattering computations in two dimensions, with current hardware. For inverse scattering problems in three dimensions, a modified ray tracing method, less accurate but faster, is more practical for the forward modeling, and will be implemented.

Analyses and Direct Methods: The stable inversion methods described in [2]–[3] are iterative procedures which make their analyses more difficult. In two and three dimensions the only direct method numerically tested is the so-called layer stripping method. It is numerically unstable; it is not even stable enough to permit convergence analysis. It turns out that there is a numerically stable way to perform layer stripping. I will finish its numerical implementation, and, since this is a direct method, wish to prove convergence.

As a final remark, two more of the proposed projects (Obstacle inverse scattering and one-dimensional layered media) have already been advanced and one of them is near completion. The Solids in Water project will receive my next fullest attention; I have been consulting with experts in this area and preparing for the effort for quite a while. We will come to close grips with the Fast Algorithm project once a suitable graduate student or post doc is available, since this activity is quite programming intensive in which not only time and patience, but also organizational skills, are needed to tackle mathematical and programming issues of complicated structures.

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